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ABSTRACT

A new model for turbulent wake mixing is proposed. The model, which appears supported by laboratory experiments and which is made plausible on physical grounds, considers the wake as consisting of a "marble cake" structure of hot and cold lumps, the cold lumps being mixed into the wake by the large eddies of the turbulent wake. A dynamical (one-dimensional) model for the mixing leads to a simple set of equations which can be numerically integrated to yield the temperature, density and velocity of the hot and cold wake components. The inclusion of chemistry in the model would yield wake composition. The model permits the prediction of mean square mass and electron density fluctuations. The fluctuations are assumed to arise from random convection of inhomogeneous fluid elements, rather than by compressibility effects or dissipation associated with the turbulent velocity field. The consequences of such mixing for the spectrum of a scalar additive in a wake are qualitatively discussed.

Comparison of predicted values of gas density fluctuation with experimental results show encouraging agreement, and warrant further development of the model.

CONTENTS

1. INTRODUCTION

The passage of a hypervelocity object through the atmosphere generates, at sufficiently high Reynolds number, a wake consisting of two distinct portions. The inner wake portion is turbulent, and relatively hotter than the outer, inviscid portion. The inner wake grows with downstream distance to engulf the outer flow, and the turbulence plays a major role in the wake growth, and in the mixing of the relatively cooler outer flow into the turbulent core. The present report represents an attempt to provide a phenomenological description of mixing of the outer wake gases with the inner core gases. It is based on experimental evidence of a coarse grained structure of the wake[1,2] and attempts to explain the large density oscillations[2] measured in wakes in laboratory measurements. It is argued that these must arise from convection, by the turbulent velocity field, of dense gas from the outer wake, into the inner wake. Fluctuations arising from compressibility effects, or from temperature fluctuations associated with the turbulent velocity field, are assumed to be small by comparison.

The main virtue of the present model lies in its simplicity, rather than in its ability to make accurate predictions. It requires to be refined and extended (particularly to include chemical reactions) before it can be meaningfully applied to obtain quantitative results for hypersonic re-entry wakes.

The motivation for the current work is the fact that an understanding of the mixing process in the wake is essential to the determination of the radar cross sections of turbulent wakes. Indeed the mixing not only determines in large part the nature of the fluctuations of the electron density in the wake, but also affects the mean electron density because of the dependence of the chemistry in the wake on the mixing.

2. STRUCTURE AND GROWTH OF THE TURBULENT WAKE

The present section is devoted to a discussion of a model for the turbulent wake core which represents the latter as consisting of two parts: a relatively hot portion of homogeneously mixed gas, and a relatively cold portion representing fluid engulfed by the expanding turbulent core, but which has not yet mixed on the molecular scale with the core gas. The physical arguments behind such a model are first discussed in Section 2.1, together with experimental evidence which may be adduced in favor of such a model. Section 2.2 then presents a simple analytical model of wake mixing which leads to the 'two-fluid' structure of the wake and introduces a 'lag-time' or 'lag-distance' for mixing of newly engulfed fluid with the remainder of the wake gas. An attempt is also made to relate this lag to kinetic energy of the wake turbulence.

2.1 Physical Model of Wake Structure

The wake of hypersonic blunt bodies consists,[5] at sufficiently high Reynolds numbers, of an inviscid outer wake and a viscous turbulent inner wake. This wake structure is well-known and need not be described here. It is sufficient to recall that the turbulent wake consists initially (i.e., at the neck) of the gas contained in the free shear layers shed from the body upon separation of the boundary layer and that it grows with downstream distance, and eventually engulfs the entire (originally inviscid) outer wake.

Experimental observations of turbulent wakes in hypervelocity ranges[3,4] and of turbulent subsonic wakes[6,7] reveal that the turbulent wake possesses a sharp front which separates it from the non-turbulent outside fluid. According to presently accepted notions in the theory of turbulence,[8] the turbulent wake consists of eddies of widely varied

sizes. Most of the kinetic energy of turbulence is contained in the relatively larger eddies, while the smaller eddies are responsible for most of the viscous dissipation of kinetic energy into heat. The very largest eddies do not contain much energy but they are important in contorting the wake surface. There is a continuous transfer of turbulence energy from eddies of a given size to smaller eddies, the rate of transfer depending on the eddy size. The large energy containing eddies have a relatively long lifetime, whereas the small energy dissipating eddies have much shorter lifetimes. In general, the eddy lifetime decreases rapidly with eddy size.

The growth of the turbulent wake and the manner in which it engulfs outer fluid may be broadly characterized as follows: The local propagation of the turbulent front bounding the wake core occurs by diffusion of vorticity fluctuations into the outer fluid.[7,8] The diffusion is initiated by the smallest eddies, since they represent the greatest velocity gradients. The transfer of turbulence energy to the previously non-turbulent fluid is accomplished by the energy containing eddies, which transfer energy to the vorticity diffused into the outer fluid by the small eddies. In the above manner, turbulence is generated into previously quiescent fluid.

The mixing of outer fluid into the turbulent core, as distinguished from the transfer of turbulence energy, probably is due to large-scale mixing by the large eddies which distort the core boundary, though they do not themselves contain much energy. Only these eddies can be expected to be effective in mixing outer fluid into the wake, the smaller eddies serving to diffuse vorticity fluctuations.

The important features of the above admittedly superficial description of the turbulent wake and its growth are simply that the outer fluid is most probably mixed into the core by large scale eddies

and that the turbulent front represents the boundary between different types of velocity fields and that consequently differences in composition or density between turbulent and quiescent fluid are not instantaneously erased as the wake engulfs the latter.

Thus, at least initially, the outer fluid is mixed into the wake in relatively large 'lumps', which contain vorticity fluctuations to a greater or lesser extent, but which retain, in particular, their initial relatively low temperature without appreciable internal fluctuations.* This gives rise to a coarse-grained structure of the turbulent wake, which has been noted previously by Herlin and Hermann,[9] for instance.** The experiments of Slattery and Clay[3,4] appear to confirm such a wake model. Indeed, the large mass density fluctuations which they measure in the wake would be very difficult to explain on the basis of a well-mixed wake. Turbulent velocity, temperature, and pressure fluctuations can be reasonably expected to generate density fluctuations of no more than a few percent, whereas the observed values range up to ninety percent.

The order of magnitude of the density fluctuations that could be generated in a turbulent wake may be estimated as follows: assume that the source of the fluctuations is dissipation or pV work and is generated by the fluctuating velocities rather than convection of cold high density gas into hot low density gas. Since there is no way to support large pressure fluctuations in a free flow like a wake, we assume that the fluctuations occur at constant pressure. For the case of oscillations generated by viscous dissipation, it may be assumed that at most all

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*Note that such a blob of constant property fluid embedded in a turbulent medium containing spatial density oscillations would not be identifiable in a shadow or schlieren photograph.

**Herlin and Hermann refer to the structure as a "marble-cake" structure.

the random kinetic energy per unit mass in a given unit mass of fluid will be converted into heat in generating a density change.*

The kinetic energy E_{turb} per unit mass in the random velocity field is approximately

$$E_{turb} = \frac{3}{2} u'^2 = \frac{3}{2} \left(\frac{u'^2}{u_w^2}\right) \left(\frac{u_w}{u_\infty}\right)^2 u_\infty^2 , \qquad (1)$$

where u' is the root mean square turbulent velocity, u_w is the wake velocity, and u_∞ is the free stream velocity.

The ratio $\left(\frac{u'^2}{u_w^2}\right)$ can be estimated to be roughly one-tenth.[6]

The wake velocity is also down to 10% or less of the free stream within 20 or so body diameters.[3,4] Thus $\left(\frac{u_w}{u_\infty}\right)^2 \lesssim 10^{-2}$.

On the other hand, assuming that the kinetic energy of turbulence is converted to heat so that $c_p \Delta T \approx E_{turb}$, the density change is

$$\frac{\Delta\rho}{\rho} \approx - \frac{\Delta T}{T} = - \frac{E_{turb}}{c_p T} = - \frac{3}{2} \left(\frac{u'^2}{u_w^2}\right) \left(\frac{u_w}{u_\infty}\right) \times 2 \left(\frac{\frac{1}{2} u_\infty^2}{h}\right) \qquad (2)$$

where h is the local static enthalpy. The ratio $\left(\frac{\frac{1}{2} u_\infty^2}{h}\right)$ at about 20 body diameters behind a blunt body at 20,000 ft/sec is approximately 8. Substituting all the above numerical estimates into Eq (2), we finally get

$$\left|\frac{\Delta\rho}{\rho}\right| \approx \frac{3}{2} \times 10^{-1} \times 10^{-2} \times 2 \times 8 \approx 2.5 \times 10^{-2}$$

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*This is an upper limit by any standard.

If we proceed further downstream along the wake, $\left(\frac{u_w}{u_\infty}\right)$ decreases, while $\left(\frac{\frac{1}{2}u_\infty^2}{h}\right)$ increases and the net result gives smaller values yet for $\frac{\Delta\rho}{\rho}$. The same approximate values result, of course, from estimating $\left(\frac{\Delta\rho}{\rho}\right)$ by assuming that the turbulence energy is converted into pV work.

A wake consisting of hot and cold lumps, on the other hand, can be expected to predict density fluctuations of the observed order of magnitude, as will be shown in Section 3.

Similarly, the observations of Slattery and Clay that the correlation length of the mass density fluctuations in wakes are of the order of a body diameter can be considered to argue in favor of a coarse-grained wake structure.

The preceding considerations and interpretations of experimental results thus indicate that a wake structure consisting of a coarse-grained structure of volumes of newly engulfed outer fluid embedded in a more homogeneous core is at least plausible. In the next sub-section, a very simple model of such a coarse-grained wake is discussed.

2.2 Simple Wake Mixing Model

In the preceding sub-section, it was argued that relatively large lumps of inviscid fluid are mixed into the turbulent wake core as the latter grows behind a body. The subsequent mixing of this entrained gas in a more intimate manner with the core gas depends on the turbulence intensity and molecular diffusion in the wake, or more precisely, on the eddy and molecular diffusion coefficients. When the intensity of turbulence is high, the redistribution of inhomogeneities in the wake is caused essentially by random convection by the turbulent field, for inhomogeneities whose scale is above the "cut-off" scale for velocity fluctuations, set by viscosity effect. The effects of molecular conduction and diffusion are negligible for inhomogeneities of such a scale. Below the "cut-off" scale of turbulence, molecular effects dominate and act to erase

small scale inhomogeneities. In the absence of molecular diffusion and conduction, the effect of eddy diffusivity (or random convection) is to break up large volume elements into smaller ones (down to the cut-off scale) and, therefore, to increase mean gradients rather than decrease them. The increase in mean gradients leads, however, to increased molecular dissipation, so that eddy diffusion indirectly leads to dissipation of inhomogeneities.

The process of mixing in the early portions of the wake where the turbulence intensity is high may therefore be approximately viewed as follows: The newly entrained, relatively large volume elements in the wake are broken up into ever-smaller elements by random convection. This break up characteristic of turbulence, proceeds without appreciable molecular effects until the size of the volume elements and the resulting gradients are such that molecular effects become important and erase the inhomogeneities, leading to a well-mixed structure. If the 'take-over' of molecular effects is sufficiently sharp, there will be a negligible fraction of gas which is neither unmixed nor completely mixed (on the molecular level, that is), and the core will consist of gas in essentially two states only, namely the unmixed state corresponding to the state of the newly entered fluid, and the mixed state, corresponding to the state of the gas which has been within the turbulent core long enough to be thoroughly assimilated.

In the far wake, where the turbulent velocities have presumably died down, the above model is not expected to hold. The eddy diffusity is probably no longer very important, and the break up of the lumps no longer occurs very efficiently. The temperature inhomogeneities then decay essentially by conduction. In fact, sufficiently far downstream all velocities will have essentially died down. Thus, the final steps of thermal equalization proceed purely by molecular diffusion.

The limits of application of the 'two-fluid' model is undoubtedly dependent on Reynolds number and body geometry, and it may not be applicable at all when the turbulent intensity is very low, corresponding to very small Reynolds numbers.

The net effect of the mixing process described above is to eventually mix fluid entering the turbulent core homogeneously with the core gas. The mixing does not occur, however, instantaneously as the fluid enters inside the turbulent front, but after some lag in time and therefore downstream distance, which depends on the intensity of turbulence and on the molecular viscosity and diffusivity in the wake. The above mixing model is therefore intermediate between the homogeneous mixing model which assumes instantaneous mixing, and the inviscid random convection model of Obukhof[10] and Corrsin,[11] in which it is assumed that molecular mixing does not occur at all. The two above extremes correspond to a zero and infinite lag respectively. A quasi one-dimensional model for (chemically reacting) turbulent wakes has been analyzed by Lin and Hayes[12] for the extreme cases of homogeneous mixing and inviscid random convection. A set of equations describing the chemical reactions and mixing in the wake for finite lags can similarly be written and constitutes a simple generalization of the work of Lin and Hayes.

2.3 Equations Describing Quasi One-Dimensional Model for Wake Mixing with Lag

The model of a wake consisting of homogeneously mixed 'old' fluid and unmixed 'new' fluid in various stages of break up can be described by a quasi one-dimensional wake model, in which the mean turbulent wake boundary is assumed to be specified. The model is illustrated in Fig. 1. The fluctuations in the turbulent front are ignored. The growth of the turbulent wake is represented by the flux

of outer inviscid fluid into the turbulent core through the wake boundary. The entering fluid then forms part of the unmixed portion of the core for a mean distance ζ (measured in body diameters), after which it is homogeneously mixed with the remainder of the wake by molecular effects. The actual mixing of fluid entering at a particular station in the wake need not occur suddenly after a distance ζ, but may be spread over a (small) range of lags centered about ζ.

The turbulent wake is then characterized by the pressure (assumed constant across the wake), and by the velocity, density, enthalpy and chemical composition (if chemistry is included) of the 'hot' homogeneously mixed and 'cold' unmixed portions of the wake, together with the relative fractions of each, as a function of downstream distance. The values of the velocity, density, enthalpy, and chemical composition of the inviscid gas at the edge of the turbulent core are assumed specified, together with the total wake width or area. The wake evolution is then described by the one-dimensional equations of conservation of mass, momentum and energy, by corresponding equations describing the mixing lag, and by the equations expressing the chemical reactions or the thermodynamic characteristics of the gas (equations of state) if chemical reactions can be ignored. In the latter case, which is applicable to wakes of relatively low-speed projectiles in ballistic ranges for instance, the equations specifying the quasi-one dimensional model are the following (z denotes downstream distance):

Conservation of Mass:

$$\frac{d}{dz}(\rho_c u_c A_c) + \frac{d}{dz}(\rho_h u_h A_h) = \rho_i u_i \frac{dA}{dz} \tag{3}$$

Conservation of Momentum:

$$\frac{d}{dz}(\rho_c u_c^2 A_c) + \frac{d}{dz}(\rho_h u_h^2 A_h) - A\frac{dp}{dz} = \rho_i u_i^2 \frac{dA}{dz} \tag{4}$$

Conservation of Energy:

$$\frac{d}{dz}\left[\rho_c u_c \left(h_c + \frac{u_c^2}{2}\right) A_c\right] + \frac{d}{dz}\left[\rho_h u_h \left(h_h + \frac{u_h^2}{2}\right) A_h\right] = \rho_i u_i H \frac{dA}{dz} \tag{5}$$

Lag Equation in Terms of Mass:

$$\frac{d}{dz}(\rho_h u_h A_h) = \rho_i' u_i' \frac{dA'}{dz} \tag{6}$$

Lag Equation in Terms of Momentum:

$$\frac{d}{dz}(\rho_h u_h A_h) - A_h \frac{dp}{dz} = \rho_i' u_i'^2 \frac{dA'}{dz} \tag{7}$$

Lag Equation in Terms of Energy:

$$\frac{d}{dz}\left[\rho_h u_h \left(h_h + \frac{u_h^2}{2}\right) A_h\right] - T = \rho_i' u_i' H \frac{dA'}{dz} \tag{8}$$

Equations of State (assuming a perfect gas):

$$\rho_c h_c = \rho_h h_h = \frac{\gamma}{\gamma - 1} p \tag{9}$$

In the above equations, ρ_i is the density of the inviscid gas at the edge of the turbulent core, u_i is its velocity, H its total enthalpy, ($H = h_i + \frac{u_i^2}{2}$), and p is the pressure. They are all assumed known, except for the pressure.

The density ρ, velocity u, and static enthalpy h, for the hot and cold portions of the turbulent wake are denoted by the subscripts h and c, respectively. The areas A_h and A_c are the 'partial areas' of hot and cold fluid, and their sum is equal to the wake area A:

$$A_h + A_c = A. \tag{10}$$

All unprimed quantities are evaluated at the station z, and all primed quantities are evaluated at the station $z - \zeta$.

Equations (4), (5), and (6) simply express the fact that outer fluid entering the turbulent wake at station $z - \zeta$ is homogeneously mixed with, and becomes part of the hot gas at station z.

The term T in Eq. (6) represents the transfer of heat between the hot and cold gases in the wake by molecular conduction. If T is set equal to zero, the energy conservation equation and its lag counterpart are greatly simplified and reduce to the equations:

$$h_c + \frac{u_c^2}{2} = h_h + \frac{u_h^2}{2} = H. \tag{5a}$$

If chemical reactions are included a set of equations describing the reactions must be used, which exhibits the feature that chemical reactions proceed at different rates before and after mixing.

The lag distance ζ must be determined in order to obtain a solution to the set of Eqs. (3) through (9), or the corresponding equations including chemistry.

The mixing lag distance ζ is approximately equal to some lag time τ multiplied by the free stream velocity u_∞, i.e., $\zeta = u_\infty \tau$. The lag time τ represents the time required for the turbulent velocity field to break down the initial lumps of inviscid fluid entering the wake to the size at which molecular diffusion takes over. Thus, it is not unreasonable to assume that τ is proportional to the characteristic time for the transfer of turbulence energy from the scale of the initial lumps to the turbulence cut-off scale.

This time is determined by the energy transfer rate $\mathcal{E}$ of turbulence energy. $\mathcal{E}$ is given by+

$$\mathcal{E} = 2\nu \int_0^\infty E(k)\, k^2\, dk \approx -\, 3/2 \frac{du'^2}{dt} \qquad (11)$$

where $\mathcal{E}$ (k) is the wave-number spectrum of the velocity fluctuations, and u'^2 is the mean square amplitude of the velocity fluctuations. τ may be taken as proportional to the (logarithmic) rate of energy transfer

$$1/\tau \sim \frac{1}{u'^2} \frac{du'^2}{dt} \approx u'/l_e \qquad (12)$$

where l_e is the scale of the energy containing eddies of the turbulence.+ The latter may be shown to be equal to a fraction of the wake width when the local flow Reynolds number is high, i.e. $l_e \approx \beta\, b_w$, where b_w is the wake width and β is a numerical factor around one third or one quarter. Thus, we have approximately

$$\zeta \approx u_\infty \tau \approx \left(\frac{u_\infty}{u'}\right) b_w = \left(\frac{u_\infty}{u_w}\right)\left(\frac{u_w}{u'}\right) b_w \qquad (13)$$

+See for instance Hinze[8] for a more detailed definition of terms, and a discussion of the results used here.

where u_w is the mean (local) wake velocity measured in a stationary (laboratory) system.

The ratio $\left(\frac{u_w}{u_\infty}\right)$ varies approximately as the two-thirds power of downstream distance (x/d), and the wake width b_w varies as $(x/d)^{\frac{1}{3}}$. Thus, ζ is approximately proportional to downstream distance along the wake when the intensity of turbulence is sufficiently high so that the eddy diffusivity is very much larger than the molecular diffusivity. For lower turbulence intensities, the lag distance can be expected to increase more sharply with downstream distance.

3. MASS DENSITY OSCILLATIONS IN THE MIXING-WITH-LAG WAKE MODEL

The mean values of the mass and electron density variations predicted by the present wake model can now be discussed. Chemical reactions will not be considered in the present discussion. The wake structure is therefore specified, at any axial distance, by specifying the fractions of mixed and unmixed fluids and by specifying their densities or temperatures. Here it is assumed, once more, that each of the two portions is characterized by a single temperature or density.

Consider a turbulent wake consisting of a granular mixture of the same fluid at a given pressure but at two different temperatures T_h and T_c.

The mean square oscillations of density in a wake consisting of a mixture of 'hot' and 'cold' gases at two densities ρ_h and ρ_c, with the volume fraction of cold gas being ϵ, has been discussed in another report[14]. The relative fluctuation was shown to depend only on ϵ and on the ratio $\tau_\rho = \left(\frac{\rho_c}{\rho_h}\right)$ of the cold to the hot densities. The value of $\left(\frac{\Delta\rho}{\rho}\right)^2$ was found

$$\left(\frac{\Delta\rho}{\rho}\right)^2 = \frac{\epsilon\,(1-\epsilon)\;(\tau_\rho - 1)^2}{[1 + \epsilon\,(\tau_\rho - 1)]^2} \qquad (14)$$

where $\Delta\rho$ is the root mean square density fluctuation, and $\bar{\rho}$ is the mean density.

For the wakes of hypervelocity objects at relatively low Mach numbers, (where chemistry in particular can be ignored) it may be assumed that the perfect gas law describes the gases reasonably well. In that case, the ratio r_ρ may be approximately found by setting

$$r_\rho = \frac{\rho_c}{\rho_h} \approx \frac{T_h}{T_c} \tag{15}$$

where T_h and T_c are the temperatures of the 'hot' and 'cold' wake components.

In order to predict $\frac{\Delta\rho}{\bar{\rho}}$ from Eq. (14), both ϵ and the ratio $r_\rho \approx \frac{T_h}{T_c}$ must be specified. A determination of those quantities for the present wake mixing model requires the solution of the set of equations (3) through (9) which describe the mixing. However, a simple estimate of the mean time value of the mass density oscillations and its variation with downstream distance may be obtained by using existing solutions[5] for the axis and edge enthalpies in hypersonic wakes based on homogeneous mixing models and some estimate for the mixing lag distance ζ.

The axis and edge temperatures can be used to represent T_h and T_c. The wake cooling in homogeneous mixing models is primarily due to the admixture of cold outer fluid as the wake grows. Thus, to a first approximation, a lag in mixing of newly engulfed gas can be expected to result in a corresponding lag in the core temperature decay. Thus, the core temperature T_h for the model of mixing with lag can be approximately obtained from the instantaneous mixing solution by 'displacing' the latter by the lag distance. The unmixed portion of the cold wake at z consists of fluid which entered the turbulent wake

between $z - \zeta$ and z. Therefore, the temperature T_c of the 'cold' portion of the turbulent wake at a given downstream location z can be approximately taken to be the mean value of the temperature of the inviscid fluid at the turbulent wake edge, averaged over a distance behind the given station equal to the lag distance ζ.

The axis and edge temperatures for the turbulent wake of a sphere traveling at Mach number 8.5, as computed by Hromas[13] are depicted in Fig. 2, together with the resultant 'hot' and 'cold' temperatures obtained by introducing a mixing lag, as discussed above. The lag distance selected in Fig. 2 is $\zeta = \beta\ (x/d)$, with $\beta = 0.3$. In other words, the lag distance z is assumed to increase linearly with z, in accordance with the discussion of the variation of ζ in Section 2. The choice of the value of the coefficient β is essentially arbitrary at this point, and in computing values of $\left(\frac{\Delta\rho}{\bar{\rho}}\right)$ for the mixing-with-lag model values of $\beta = 0.1$ and $\beta = 0.3$ have been chosen (cf Fig. 3).

The value of ϵ at any station is determined by the ratio of the volume of gas which entered the turbulent boundary between z and $z - \zeta$ to the volume of gas present in the wake at z. If the wake growth is known, ϵ may be computed approximately as

$$\epsilon \approx \frac{A\,(z) \; - \; A\,(z - \zeta)}{A\,(z)} \tag{18}$$

where A is the mean wake cross section, $A = \frac{\pi b_w^2}{4}$ (b_w is the wake diameter). Using a one-third power law fitted to experiment to obtain A (z) and using the hot and cold temperatures obtained as discussed previously, the values of $\left(\frac{\Delta\rho}{\bar{\rho}}\right)$ predicted by the wake mixing mode for the case of the 8.5 Mach number sphere wake. Also shown in that figure are experimental determinations of $\left(\frac{\Delta\rho}{\bar{\rho}}\right)$ obtained by Slattery and Clay[4] for very nearly the same velocity. It may be noted that the theoretical predictions give good agreement in terms of the shape of

the $\left(\frac{\Delta\rho}{\rho}\right)$ curve, although they do not predict the experimental amplitudes and are flatter than the experimental curve. Note also that the maxima in all the theoretical curves occur virtually at the same downstream location, which is very close to the apparent maximum of the experimental curve. The agreement is regarded as quite encouraging, considering the crudeness of the computations.

4. SPECTRUM OF ELECTRON DENSITY FLUCTUATIONS

The preceding discussion of the wake mixing model, and the considerations of a previous report[14] were concerned with the mean value of mass or electron density fluctuations in a wake. The radar cross section of a wake or portion of a wake depends, however, on the statistics of the electron density fluctuations, and not simply on their mean intensity.

The wake gas and the electrons within it are convected by the random velocity fluctuations. The electron density fluctuations and the turbulent velocity fluctuations are therefore undoubtedly related. The relation between the electron density and velocity fluctuations and in particular the spectrum of electron density fluctuations generated by the turbulence must be known for application to radar cross-computations.

The first step towards determining the electron density fluctuation spectrum is a clear understanding of the role of turbulence in generating and convecting electron density inhomogeneities. At this time only qualitative remarks can be made regarding the generation of electron density fluctuations.

There are two basically different mechanisms which can be assumed to be responsible for the electron density fluctuations. They

can be assumed to arise primarily from the mixing, by random convection, of the outer wake gas, which is relatively cold and electron-poor with the inner turbulent core which is more highly ionized. Thus, fluctuations would be the result of a random mixing or stirring process. They could, on the other hand, be assumed to arise because of the fluctuations in pressure, temperature and density which accompany turbulence in a compressible fluid. It was shown above, however, that the latter hypothesis would be very hard pressed to explain the large density fluctuations in wakes observed experimentally in ballistic range measurements. Consequently, it will be assumed that electron density fluctuations arise from random convection.

In the random convection model, the physical process of generation and decay of electron density inhomogeneities may be understood by considering separately the effects of random convection on one hand and of diffusion and reaction on the other. Random convection acts upon the fluid newly engulfed in the turbulent core to intersperse it with the fluid already present there. If the velocity field below a certain scale is isotropic, it will tend to generate an isotropic field of fluctuation of a passive (with respect to velocity) additive such as electron density, below that scale. In particular, if the velocity spectrum is of the Kolmogoroff type, with a -5/3 power wave number dependence, it will generate, in equilibrium, a corresponding Kolmogoroff spectrum in the scalar additive. The longer the turbulent velocity field acts on the additive, the closer the distribution of additive will be to its asymptotic spectral form. On the other hand, 'new' fluid is continuously being engulfed in the turbulent core, with an initial distribution which is certainly different than the asymptotic limit to which it tends when in equilibrium, so to speak, with the velocity field.

The extent to which the turbulent velocity field generates a random electron density field (neglecting so far diffusion and reaction) independent of the initial distribution depends on the proportions of 'well-mixed' and poorly mixed fluids, and therefore, on the time scale required for the random convection to 'dissipate' the initial conditions, compared to the rate of entrainment of new fluid. It also depends on the scale of turbulence with which one is concerned. Thus, the large scale inhomogeneities cannot be expected to exhibit an 'asymptotic' spectrum, since they arise from the initial mixture of fluid in the wake. On the other hand, the inhomogeneities on some small enough scale would undoubtedly be isotropic and characteristic only of the velocity fields and not of the initial conditions of mixing. However, for the small scale inhomogeneities, the effects of diffusion become quite important.

We may conclude from the above discussion that, in the absence of diffusion, the spectrum of a passive additive in the turbulent wake core would be controlled by the generation of large scale inhomogeneities by the wake growth process and the 'competing' redistribution of these inhomogeneities into some 'equilibrium' structure characteristic of the velocity fluctuation spectrum by random convection. The effect of diffusion is to reduce the density gradients and therefore reduce the amplitude of the fluctuations throughout the spectrum. The effect of diffusion is strongest for the highest wave number range, both because a given absolute density difference represents a larger gradient for the smaller scales of the fluctuation, and because the relative rate of molecular diffusion compared to eddy diffusion (i. e., random convection) increases rapidly with wave number. The effects of diffusion become increasingly important and establish a 'cut-off' for the electron density

fluctuation spectrum. Whether any portion of the spectrum above the cut-off is close to the equilibrium spectrum characteristic only of the velocity field probably depends on the intensity of turbulence and on the diffusion and viscosity coefficients in the wake. If such a portion of the spectrum, located between the large scale limit characterized by the initial mixing conditions and the small scale limit set by diffusion, exists, its importance for the wake scattering problem depends on whether the radar can 'see' that portion of the spectrum. The parameters which must be known to determine whether a quasi-equilibrium isotropic electron density fluctuation spectrum characteristic only of the velocity fluctuations exists, and describes the bulk of the wake inhomogeneities in a given wave number range are essentially the mean time required for random convection to break down a 'lump' of fluid to the scale of interest and redistribute it, the time required for homogeneous molecular mixing of newly engulfed fluid into the main core fluid, and the rate of entrainment of new fluid into the core, which also controls the proportion of fluid which is describable by the equilibrium spectrum. A quantitative evaluation of those times and the determination of the quantitative dependence of the electron density fluctuation spectrum at a given wave number on those times has not been performed to date. Consequently, nothing definite can be said at this time about the electron density fluctuation spectrum in any wave-number range. Even less can be said about the more general space-time fluctuation spectrum, which determines the character of the radar cross section fluctuations (as distinguished from the mean cross-section). The determination of that spectrum and its dependence on the convection and diffusion processes in the turbulent wake is very important for the prediction of radar scattering from wakes. In the absence of a

firm understanding of the electron density fluctuations in a wake, one may attempt to predict radar cross-sections based on some plausible conjectures. However, one should be cautious about accepting such estimates, even though they may appear to agree in order of magnitude with a limited amount of experimental data.

5. CONCLUDING REMARKS

In the preceding sections, a simple model for the turbulent wake in which the latter exhibits a granular structure consisting of a mixture of 'hot' and 'cold' elements of fluid of various sizes was investigated. Such a structure was justified on the basis of physical arguments concerning the growth of the turbulent wake by engulfment of the neighboring inviscid wake gas. The wake mixing was described in terms of a mixing lag distance ζ which represents the mean distance traveled by fluid entering the turbulent boundary before it is homogeneously mixed with the turbulent core gas. The model appears to be at least partly supported by the experimental evidence of large mass density oscillations. Indeed, the model predicts a variation with downstream distance of the amplitude of the mass density oscillations in the wake quite similar to that observed experimentally.

In view of the initial encouraging results of the model, its further development and generalization appear worthwhile. The most straightforward extension of the model is the inclusion of chemical reactions. However, some firmer estimate of the mixing lag is required before calculations of wake chemistry are undertaken. Such an estimate must be based on a more detailed understanding of the roles of random convection and diffusion in the mixing process, and of the dependence of the mixing on the turbulence intensity, and therefore on the Reynolds number and body shape.

Similarly, the energy transfer between the hot and cold portions of the wake, must be carefully analyzed. More fundamentally, a generalization of the present model which relaxes the sudden mixing limitation while retaining the concept of a mixing lag should be investigated.

Finally, the nature of fluctuations predicted by the present model must be determined if application is to be made to radar scattering. The distribution of a scalar by the combined action of random convection and diffusion is known (however incompletely) only for the case of a homogeneous scalar field. The predicted distributions apply to the 'equilibrium' asymptotic case in which the effects of the initial distribution of the scalar quantity have disappeared. The modification in the fluctuation spectrum arising from the existence of mean variations and from the continuous mixing of outer fluid in the wake must be investigated.

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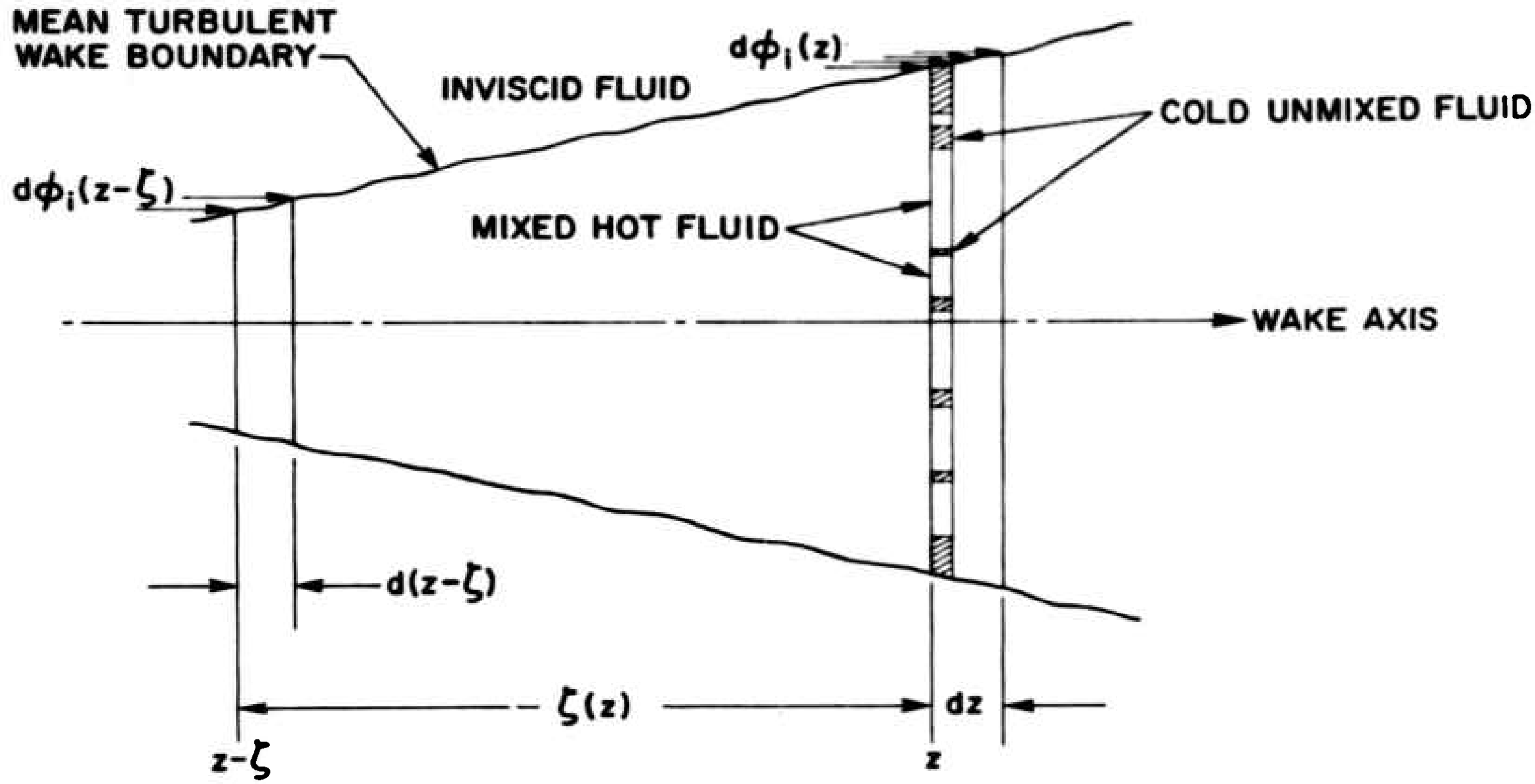

Fig. 1 Wake Mixing with Lag. The fluid entering the mean turbulent boundary at $z - \zeta$ travels a distance ζ before mixing on the molecular level with the gas already present in the wake core. Thus the change in the core density and temperature over the distance dz is controlled by the fluid entering the turbulent core within $d(z - \zeta)$. The lag in mixing leads to a turbulent core composition consisting of a 'hot', homogeneously mixed portion, and a 'cold' unmixed portion.

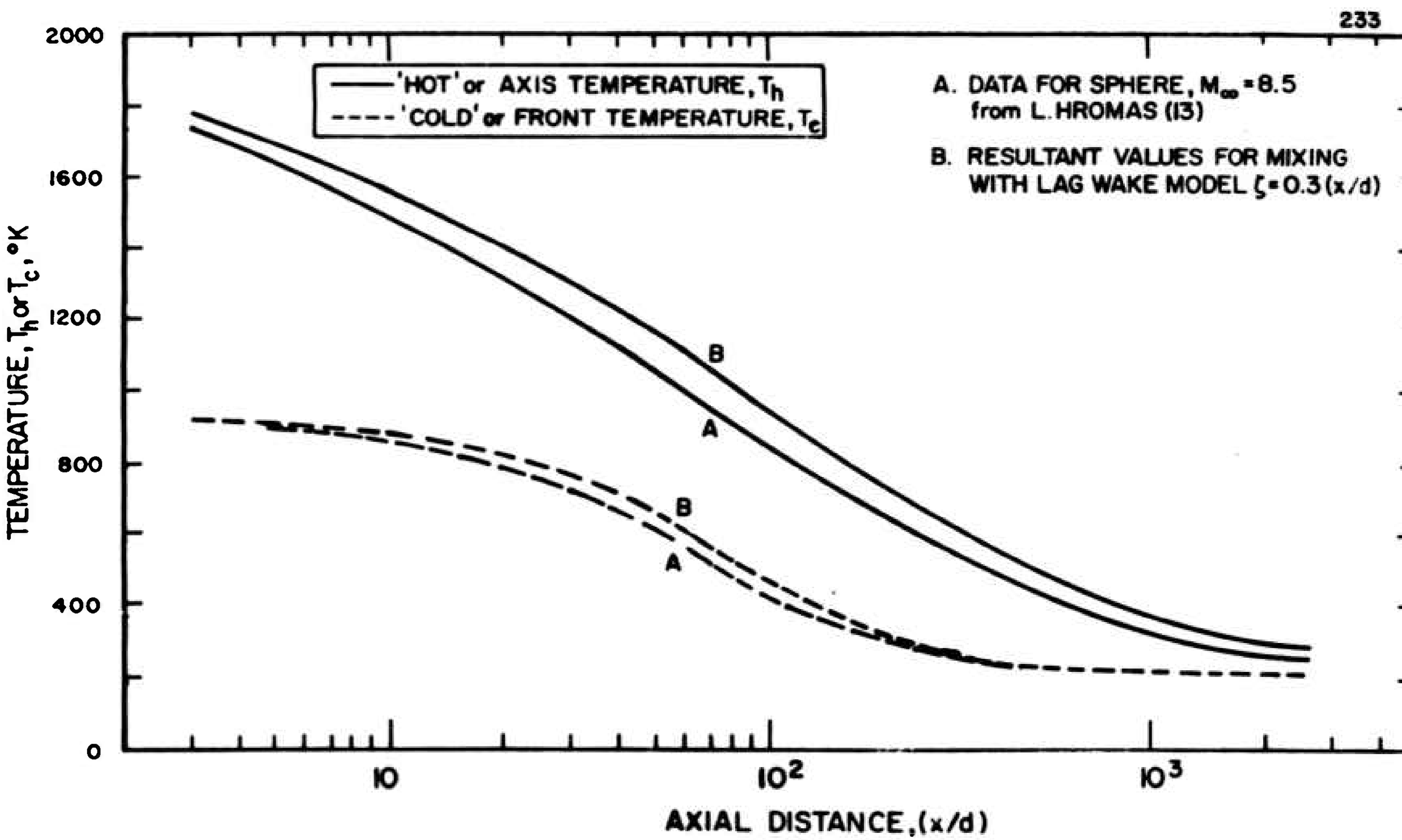

Fig. 2 Equilibrium temperatures along the axis and front of the turbulent wake of a sphere computed according to a homogeneous mixing model and corresponding 'hot' and 'cold' temperatures for the mixing-with-lag wake model.

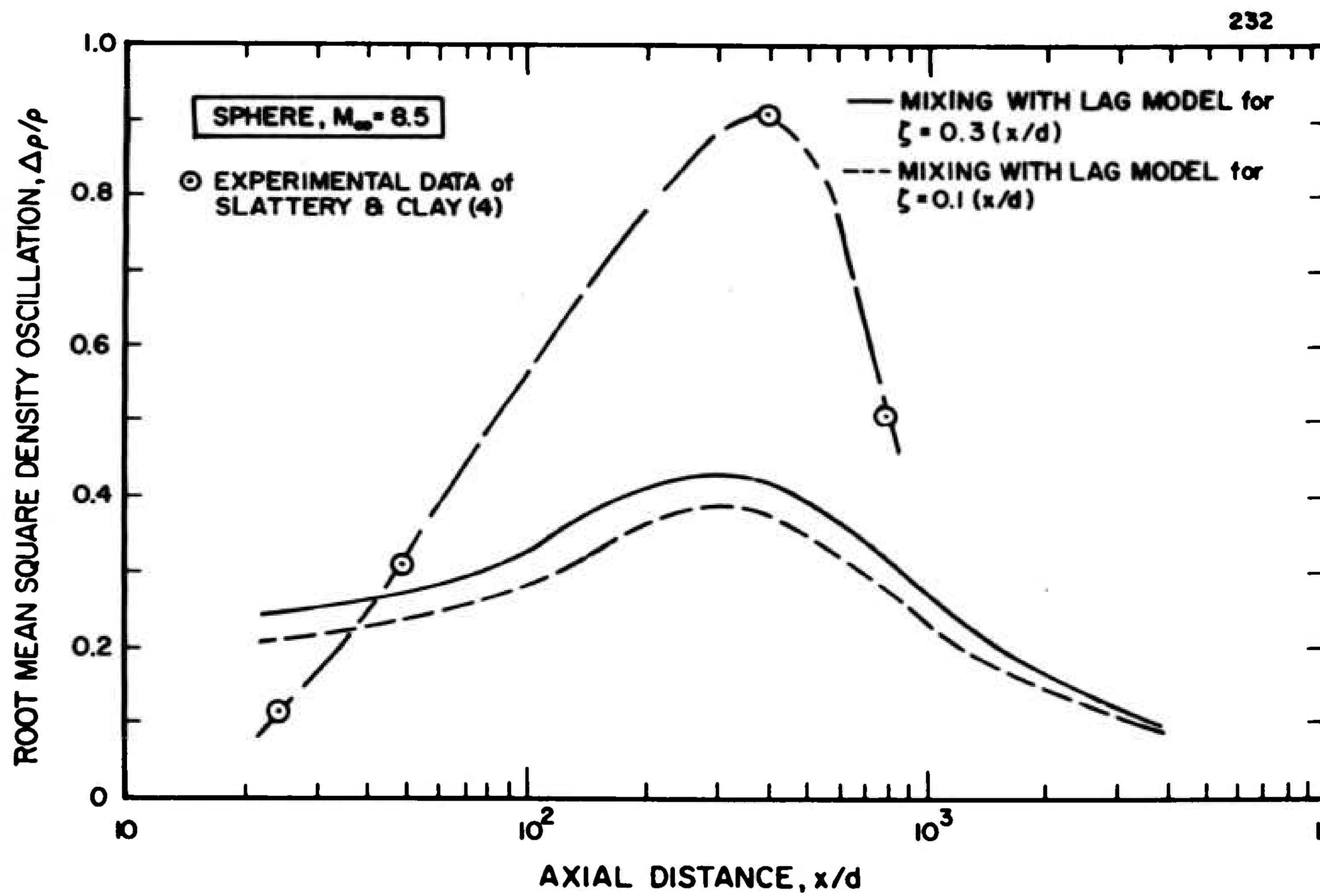

Fig. 3 Root mean square density oscillations predicted by mixing-with-lag wake model for a sphere at flight Mach number 8.5 and comparison with experiment.

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